Tips and Tricks: Evaluating Limits on a Graphical Calculator
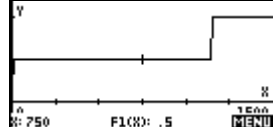
Graphical calculators such as the HP38G or HP39G can easily be used to evaluate some limits on an 'experimental' basis using tabular view.

For example, in **Function** enter **F1(X)=SIN(X)/X**. Ensure that you are in **Radian** mode and then, in **NUM SETUP** view, change the **NumType** to '**Build Your Own**'.

If you now go to the **NUM** view then you can enter values which are successively closer to zero and see quite effectively that $\lim\limits_{x\to 0}\dfrac{\sin x}{x}$ seems to be zero. Theoretical justifications can then follow of course.

However, there are some traps which lie in wait for the unwary! For example, suppose we examine $\lim\limits_{x\to\infty}\dfrac{e^x}{2e^x+5}$, which we know should have a limit of 0.5. If we use the same technique as before then everything is fine until we enter values for x larger than 1152 (see screen shot), whereupon there is an abrupt transition which can be seen even more clearly in the **PLOT** view.

The reason for this strange behavior lies with the value of **MAXREAL** which, at $9\cdot999999\times10^{499}$, is the largest number that can be manipulated on the calculator. When the calculator evaluates the fraction in this particular limit, the top and the bottom of the fraction both exceed **MAXREAL** for values of x past 1152.

This means that the fraction evaluates to $\dfrac{MAXREAL}{MAXREAL}$, which in turn evaluates to 1. An error of this type is sometimes known as a precision error.   This particular trap caught a great deal too many students in a recent state examination paper at our school in Western Australia and indicated that teachers are teaching students to use their calculators to evaluate limits without making them aware of the limitations.

Along similar lines, one of the common limits evaluated by teachers is $\lim\limits_{x\to\infty}\left(1+\dfrac{1}{x}\right)^x$ which results in the value of 'e'. Producing a PLOT of the function F1(X)=(1+1/X)^X gives a clear indication that the value is approaching a limit from below (see right) but it is a very slow convergence if you want a value which is accurate to more than a few decimal places. However, problems can arise with students who decide to zoom out to look at very large numbers and find that strange things start happening around $X=10^{11}$.
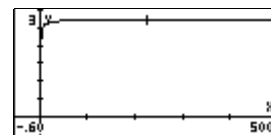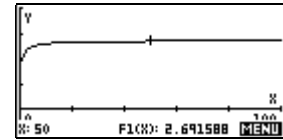
The problem is caused by the internal rounding of the calculator. It works internally to 16 bits, which means that it can store 12 significant digits (for reasons only of interest to programmers). This means that when you invert a really large number and add it to one, you lose a lot of accuracy.

For example: if X = 2.85E10 then 1/X is 2.5087719298E-11 However, when you add 1 to this, the calculator is forced to discard all but the last decimal place. Thus 1 + 1/X = 1.00000000003 (rounded off from 1.00000000002508...)
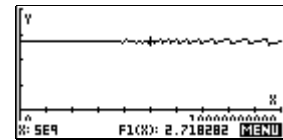
Expected result

(0 to 100)

There are naturally a whole range of numbers which will all round off to the same value of 1.00000000003, so that (for that range of numbers) the expression (1+1/X)^X is equivalent mathematically (on the HP38G) to (1.00000000003)^X. This produces a short section of an exponential graph, which only looks linear because you don't see enough of it.

Starting to be unstable

(0 to 1E11)

Eventually the calculator reaches a value on the x axis which is large enough that it rounds off to a smaller number than 1.00000000003, which is 1.00000000002 This produces the sudden drop in the graph as the plot changes from a section of a 1.00000000003^X graph to a section of a 1.00000000002^X graph (which has a shallower gradient). This section is then maintained until the next drop, and so on.
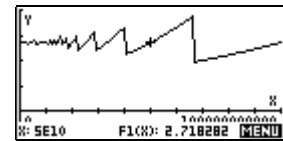
Big problems start...

(0 to 1E11)

Finally the value of X becomes so large that 1 + 1/X cannot be distinguished from 1 to the calculator's accuracy. At that point the expression (1 + 1/X)^X becomes equivalent to 1^X, which is simply a horizontal line of F(X)=1. If you change the scale from XRng 0 to 1E11 to an XRng of 0 to 4E11 you will see this happen. The point of change is when 1/X becomes less than 1E-12 when rounded, which is at X=2E11.
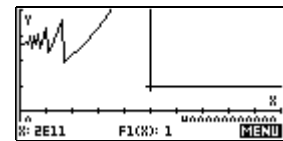
The final stages

(0 to 2E11)